

# Arbeitsbericht NAB 09-23

**Glacial erosion: a review of its  
modelling**

November 2009

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### KEYWORDS

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subglacial system, abrasion, quarrying, meltwater erosion,  
coupled ice-sheet/sediment models

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## Preface

In many countries, including Switzerland, deep geological disposal has been identified as the method of choice for the long-term management of radioactive waste. This method involves placing the suitably packaged waste in tunnels excavated in a stable rock formation with properties favourable to radionuclide retention, deep underground (deep geological repository). The underlying idea is that the radionuclides in the waste will remain contained and isolated from the accessible human environment by natural and engineered barriers until they have decayed to such a degree that the remaining radioactivity does not present an unacceptable hazard to humans or the environment.

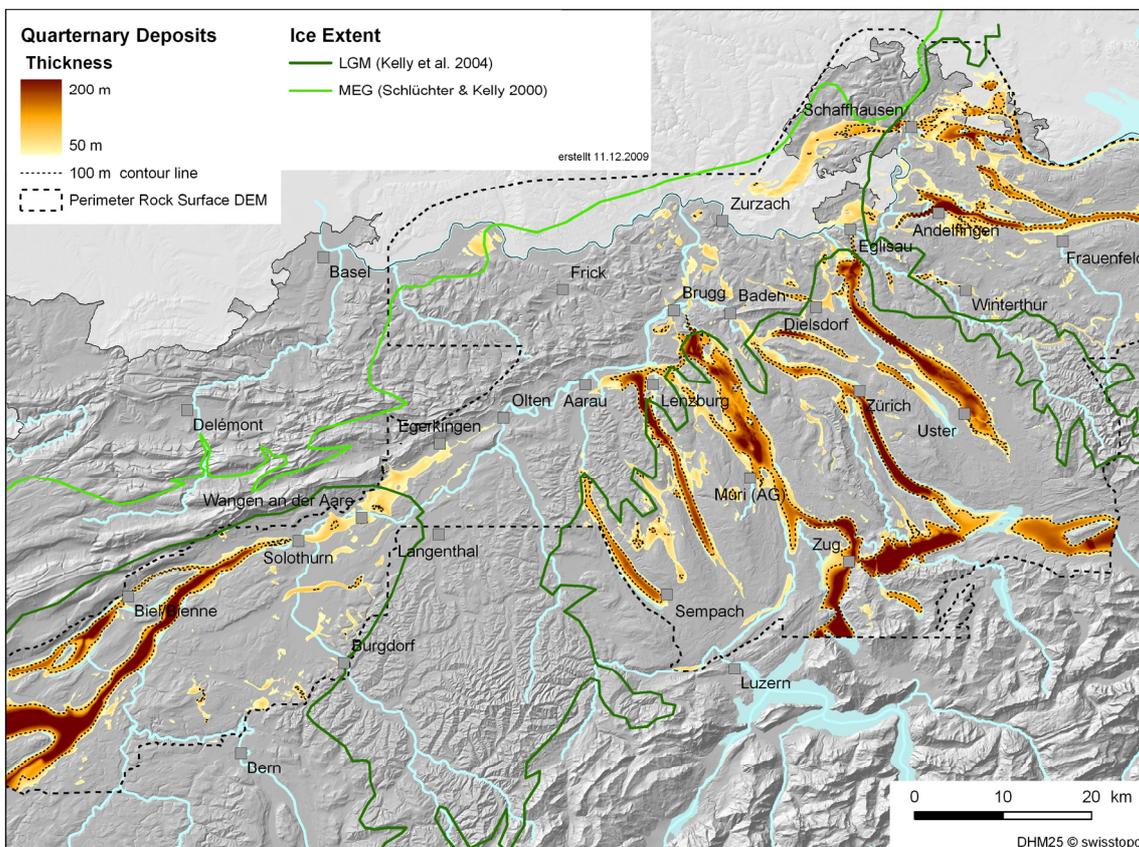


Fig. 1: System of deep, overdeepened and buried Quaternary valleys in the midland areas of northern Switzerland (modified from Nagra 2008a).

Mapping is based on information from several thousand boreholes and other sources. Also shown is the ice extent at the Last Glacial Maximum (LGM) and during the Most Extensive Glaciation (MEG).

The National Cooperative for the Disposal of Radioactive Waste (Nagra) is developing concepts for implementing deep geological repositories in Switzerland and for performing the necessary analyses of the long-term safety that they can offer. As part of these safety analyses the geomorphological evolution of the landscape has to be evaluated for a time horizon of one million years for high level waste. One of the relevant aspects concerns the effects of “deep glacial erosion” on the long-term safety of a geological repository. This term refers to the origin

of deeply incised troughs and overdeepened valleys (Nagra 2008a) beneath glaciers that extended from the Alps far into the Alpine foreland and covered the midland areas of northern Switzerland (Swiss Plateau) with hundreds of meters thick ice several times during the Quaternary (Fig. 1).

As the climate in northern Switzerland in the time period of concern (1 Ma) is likely to continue to oscillate between glacial and interglacial periods, the question as to when, where and how often future glaciations can lead to deep glacial erosion is of great importance for the long-term disposal of radioactive waste. Of significance in this respect is that all of the proposed geological siting regions (Nagra 2008b) are located within the ice extent during the Most Extensive Glaciation (MEG), some of them even within that at the Last Glacial Maximum (LGM).

While the glacial erosion processes involved in the formation of a glacial topography are generally known, the processes that lead to deep glacial erosion beneath the frontal reaches of glaciers far away from alpine-type mountain ranges remain incompletely understood. Furthermore, the factors that control processes and erosion rates beneath the ice are a matter of debate. In this context, broad overdeepenings resulting from large-area denudation by subglacial abrasion are generally distinguished from slot gorges associated with linear down-cutting of subglacial meltwater channels. Much controversy is also centered on the question whether the deeply incised troughs and overdeepened valleys in the midland areas of northern Switzerland were primarily carved out during a single glacial cycle or eroded and deepened in successive glaciations. Another important issue relates to the morphological control on the flow of advancing ice at the onset of glaciations and thereby the focussing of future erosion in already existing overdeepened valleys.

Besides reconstructing the history of glaciations in the midland areas of northern Switzerland by determining the Quaternary deposition history, numerical modelling of glacial erosion can help to obtain a better understanding of the processes governing past and future subglacial landform evolution. The aim of this report is to give an overview of existing modelling approaches as a basis for discussing necessary model developments and code adjustments with respect to glacial erosion in distal alpine-type forelands.

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# 1 Introduction

Glaciers and ice sheets are important agents of erosion, transport and deposition and thus constitute an influential component in the evolution of the landscape which they cover. At the same time, glaciers and ice sheets respond dynamically to changes in boundary conditions, such as climatic variations and topographic adjustments. With the advent of numerical modelling techniques and recent advances in computer technology, modelling ice-sheet dynamics has become a powerful means to investigate the complex interactions between ice masses, the climate system and subglacial landforms in a quantitative way, in past as well as future environments.

Numerical models of ice flow have been applied to the existing ice sheets of Greenland and Antarctica, and those which covered the continents of the Northern Hemisphere during the Quaternary ice ages. Typical studies have concentrated on mechanisms and threshold of ice-sheet inception (e.g. Huybrechts 1994; DeConto & Pollard 2003a), ice-sheet form and extent during glacial–interglacial cycles (e.g. Marshall et al. 2000, 2002; Ritz et al. 2001; Charbit et al. 2002) and the response of the polar ice sheets to future climatic warming (e.g. Huybrechts & de Wolde 1999; Van de Wal et al. 2001). Related studies have considered ice sheets as sources of changes in surface loading for isostasy and gravity models (e.g. Le Meur & Huybrechts 2001; Tarasov & Peltier 2004) and changes in freshwater fluxes for ocean models (e.g. Schmittner et al. 2002; Fichfet et al. 2003). By accounting for interactions between thermal and flow regimes, ice-sheet models have also been used to investigate the potential for internally generated flow instability (e.g. Payne 1995; Payne & Dongelmans 1997; Marshall & Clarke 1997). In addition, models of the Greenland and Antarctic ice sheets are in use to assist with the dating and interpretation of ice cores (e.g. Greve 1997; Huybrechts et al. 2007).

In this review the emphasis is focused on the capability of glaciers and ice sheets to erode, transport and deposit substantial quantities of sediment. The discussion that follows concentrates on models of ice flow that account for sediment erosion beneath glaciers and ice sheets. First, the main processes of glacial erosion and fundamental concepts of ice-sheet modelling are briefly introduced before various coupled ice-sheet/sediment models are presented and discussed.



## 2 Background to glacial erosion

The main processes of glacial erosion responsible for the landforms associated with glaciation are abrasion and quarrying and were identified over a century ago (e.g. Forbes 1843; Tyndall 1864). While abrasion involves the wearing down of rock surfaces by the grinding effect of glacier ice charged with basal debris, quarrying is the removal of well jointed or loosened blocks of bedrock by the overriding glacier. Significant wear to the glacier bed can also be caused by subglacial fluvial action, particularly where sediment can act as an erosive tool, but also through chemical dissolution as water comes into contact with soluble minerals in bedrock and sediment.

Glacial abrasion is the process by which rock fragments entrained in the basal layer of a glacier are dragged across the glacier bed. The scratching and polishing associated with abrasion tends to create smooth rock surfaces, often with striations and grooves. Quarrying, or plucking, is the process whereby a glacier removes blocks of bedrock of its bed. The fracturing of the rock is controlled by the density, spacing and depth of pre-existing joints in bedrock, together with the stresses applied by the glacier. Rock fracturing is also aided by the presence of meltwater beneath the glacier, where bedrock is loosened by subglacial water pressure fluctuations. Once plucked the blocks are entrained in the basal ice and become the grinding tools that cause abrasion. For abrasion and quarrying to take place, there must be some relative movement between the glacier sole and the bed which therefore requires either water at the bed or the basal ice to be at the pressure melting point. Abrasion and quarrying, therefore, are dependent upon the temperature and velocity of the basal ice, debris concentrations, ice thickness and subglacial water pressure (e.g. Boulton 1974, 1979; Hallet 1979, 1981, 1996; Iverson 1990, 1991, 2002).

Mechanical erosion of bedrock in subglacial meltwater channels proceeds in a manner similar to the denudation by sub-aerial streams exhibiting open-channel flow. Abrasion takes place by the striating or grooving of the channel surface by sediments in suspension while rock fragments may be gouged out by the impact of the coarse sediment fraction that does not become suspended but rolls, slides and saltates along the channel bottom. Mechanical meltwater erosion, therefore, is dependent upon water velocity, discharge and turbulence as well as the quantity of suspended sediments and bedload in traction (e.g. Drewry 1986; Hambrey 1994; Bennett & Glasser 2009; Benn & Evans 1998). Of specific importance in subglacial environments is that high water velocities and discharges require strong pressure gradients in hydraulically well-conducting subglacial channels.



### 3 Background to ice-sheet modelling

All models assume that the balance of forces is static, i.e., that acceleration is not significant in the balance. In zero-order models all the gravitational driving stress is balanced locally by vertical shear stresses and basal drag. This so-called shallow-ice approximation (Hutter 1983) recognizes the disparity between the vertical and horizontal length scales of ice flow and implies ice deformation by simple shear by neglecting transverse and longitudinal strain rate components. This assumption is generally applicable in large-scale ice-sheet modelling as long as surface slopes are evaluated over horizontal distances an order of magnitude greater than the ice thickness.

At the heart of three-dimensional thermomechanical models is the simultaneous solution of two evolutionary equations for ice thickness and internal temperature, together with representations of ice velocity components (e.g. Huybrechts 1990; Ritz et al. 1997; Payne 1999; Marshall et al. 2000). These express fundamental conservation laws for momentum, mass and heat, supplemented with a constitutive equation to convert the predicted stress field to a velocity field. Most commonly used in ice-flow modelling is Glen's flow law (Glen 1955), an empirical relation derived from laboratory tests. It considers ice as a non-Newtonian viscous fluid, relating strain rates to stresses raised mostly to the third power. However, in ice the rate of deformation for a given stress is also a strong function of temperature. The flow-temperature coupling is accounted for by a temperature dependence of the rate factor in Glen's flow law. Further, the inclusion of heat conduction in the underlying bedrock gives rise to a variable geothermal heat flux at the ice-sheet base depending on the thermal history of ice and rock.

Ice flow is assumed to result both from internal deformation and from basal sliding over the bed in those areas where the basal temperature is at the pressure melting point and a lubricating water layer is present. Ice deformation results from vertical shearing most of which occurs near the base. For the sliding velocity, a generalized relation that takes the effect of subglacial water pressure into account (e.g. Budd et al. 1969; Iken 1981; Bindschadler 1983; Iken & Bindschadler 1986) is usually adopted.

In recent ice-sheet models the isostatic adjustment of the bedrock to the varying ice load is taken into account by treating the bedrock as a rigid elastic plate (lithosphere) that overlies a viscous asthenosphere (e.g. Huybrechts & de Wolde 1999). This means that the isostatic compensation not only considers the local load, but integrates the contributions from more remote locations, giving rise to deviations from local isostasy.

Interaction with the atmosphere in large-scale ice-sheet models is achieved by prescribing the climatic input, consisting of surface mass-balance (accumulation minus ablation) and surface temperature. Changes in these fields are usually heavily parameterized in terms of air temperature. Precipitation rates are often based on their present distribution and perturbed in different climates according to temperature sensitivities. Surface melt is usually calculated from the positive degree-day method (e.g. Braithwaite & Olesen 1989; Braithwaite 1995; Ohmura 2001), an index method providing the bulk melting rate depending on air temperature only.

During the 1990s, the European Science Foundation (ESF) European Ice Sheet Modelling Initiative (EISMINT) programme allowed intercomparison of thermomechanical ice-sheet models to assess how well these models simulate basic geophysical variables such as ice thickness, velocity and temperature. All of the models that took part have been demonstrated to behave conformably in a comprehensive series of benchmark tests (Huybrechts et al. 1996; Payne et al. 2000). Models of this type give favourable reconstructions of present-day ice

configuration in East Antarctica and Greenland (e.g. Huybrechts 1990; Ritz et al. 1997; Marshall et al. 2000). This outcome of EISMINT is an appropriate level of regulation in ice sheet modelling activity.

## 4 Modelling of subglacial erosion

A number of studies have used one- and two-dimensional numerical models to investigate where erosion occurs beneath a glacier (Oerlemans 1984; Harbor et al. 1988; Harbor 1992, 1995; MacGregor et al. 2000, 2009; Kessler et al. 2008). Modelling by Oerlemans (1984) was the first numerical effort aimed at understanding the development of longitudinal valley profiles by glaciers. He finds that the formation of overdeepenings is favoured by climatic conditions that allow the warm-based snout of the glacier to stay roughly in the same position for a long time. In addition, numerical experiments indicate that the pre-existing topography is critical in focussing erosion as minor valleys may develop into deep glacial troughs with large ice discharges.

Models of U-shaped valley cross section development indicate that under constant ice discharge conditions, it is possible to produce a steady-state U-shaped valley from an initial V-shaped valley in a single 100 ka glacial cycle (Harbor et al. 1988; Harbor 1992, 1995). The constriction near the bottom of the V-shaped valley impedes the flow so that there is a central minimum in the basal velocity distribution. With an erosion law dependent on basal velocity, this results in erosion rates that peak on the valley sides rather than at the centre which effectively broadens the valley bottom and steepens the valley walls into the U-shaped form.

The modelling efforts of MacGregor et al. (2000, 2009) suggest that quarrying is most effective on the steepest bed slopes near the head of glaciers while rates of abrasion reflect the pattern of integrated ice discharge. The simulations show that erosion maxima occur just below the headwall as a result of enhanced quarrying there. Hence headwalls increase in height and retreat headward with time. In contrast, abrasion shows a maximum where the ice thickness is greatest. As such, steps and overdeepenings in valley longitudinal profiles develop immediately below the junction of tributary glaciers as a result of an increase in ice discharge which is accommodated primarily by increased ice thickness and hence sliding rate.

Modelling studies of fjord incision through a coastal mountain range indicate that topographic funnelling of ice and erosion proportional to ice discharge are sufficient to form fjords (Kessler et al. 2008). Well-developed fjords appear in the simulations after  $\sim 1$  Ma owing to a robust positive feedback initiated by ice being funnelled through mountain passes. Enhanced erosion beneath thicker and faster ice deepens these passes which further amplifies the topographic funnelling and causes ice flow to strongly converge towards deep fjords.

### 4.1 Landscape evolution under ice sheets

Jamieson et al. (2007) use the GLIMMER (General Land Ice Model for Multiply Enabled Regions) ice sheet model with an erosion component to examine the evolution of landscapes under ice sheets over long time scales in hypothetical situations (Fig. 2). GLIMMER is a three-dimensional thermomechanical ice sheet model which has led from developments by Boulton & Payne (1992) and Payne (1999). The model has the ability to predict ice thickness, the distribution of areas at pressure melting point and basal velocities. The erosion rate is calculated as a simple linear function of the basal velocity and therefore erosion only occurs where the bed is not frozen and basal sliding is possible. In addition, sliding velocities are assumed to be related to basal water production thereby approximating the tendency for increased basal water pressure and thus ice-bed decoupling to occur with increased basal melt.

Results suggest that erosion may be influential in stabilising the thermal regime of an ice sheet. As existing valleys are overdeepened they are more likely to become the permanent location of warm-based ice. Further, the pattern of erosion that occurs when an ice sheet grows upon a purely fluvial system is somewhat different to what would be expected had the system remained fluvial. Sediment is excavated low down in the valleys where ice is able to warm up and flow at higher velocity (Fig. 3). In a fluvial system, where slope failure and river incision are the main erosive agents, denudation occurs higher in the catchments where slopes are steeper and rivers have higher energy. Finally, after only 100 ka of glaciation at conservative erosion rates, Jamieson et al. (2007) were able to show that a preglacial fluvial landscape can display some recognisable elements of a glaciated landscape – particularly in the lower reaches of valley longitudinal profiles (Fig. 3C).

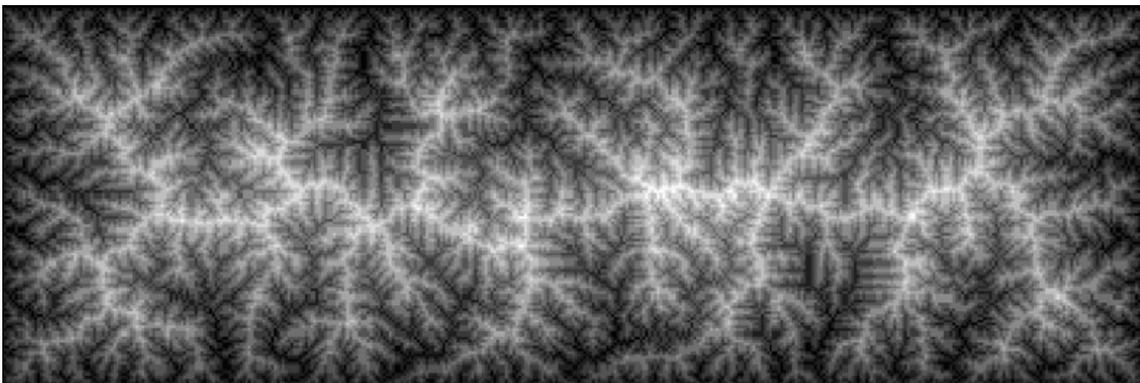


Fig. 2: Ridge-like fluvial topography generated by the GOLEM (Geomorphic/Orogenic Landscape Evolution Model) fluvial/tectonic evolution system (taken from Jamieson et al. 2007).

Elevations range from 0 m (black) to 4000 m (white).

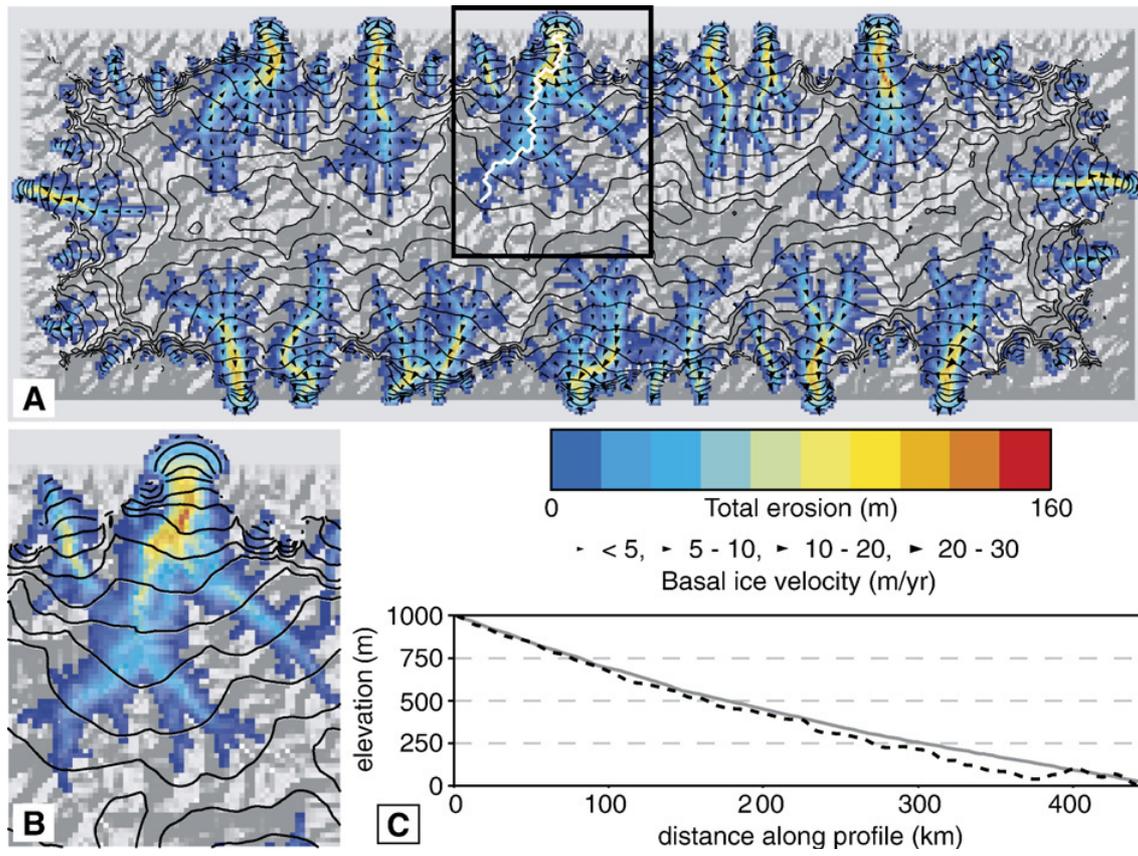


Fig. 3: Results from glaciated GOLEM topography shown in Fig. 2 (taken from Jamieson et al. 2007).

(A) Pattern of erosion after the modelled time period (100 ka) along with ice thickness (contour lines) and direction and magnitude of ice flow (black arrows). Position of valley longitudinal profiles shown in C are marked by a white line. (B) Close-up of pattern of erosion across one major basin. Position is shown by black box in A. (C) Longitudinal profiles of preglacial fluvial valley (grey) and post-glacial valley (dashed black).

## 4.2 Relief generation by polythermal glacier ice

Staiger et al. (2005) apply the University of Maine Ice Sheet Model (UMISM) to the Torngat Mountains situated on the Labrador Peninsula, north-eastern Canada to characterize the spatial variation and magnitude of erosion over a glacial cycle on a mountain-range scale. UMISM is based on a map-plane model of ice flow (Fastook & Chapman 1989; Fastook 1994; Fastook & Holmlund 1994; Fastook & Prentice 1994) which has been modified to include a calculation of basal melt water (Johnson & Fastook 2002). The model consists of a finite-element solution of the time-dependent continuity equation for mass conservation governing ice-sheet dynamics. By accounting for the geothermal, frictional and deformational heat sources the thermal dynamics of the model result in the production of melt water. In the flow simulations basal sliding velocity is taken to be controlled by the basal water depth. To obtain spatial information on potential glacial erosion beneath the ice, basal sliding distance is computed by multiplying the sliding velocity with the period that the velocity persisted (Näslund et al. 2003). Because sliding distance does not record the velocity of the ice directly, Staiger et al. (2005) normalize the simulated basal sliding distance by the duration of ice cover (Fig. 4) to distinguish areas of the bed affected by short-lived, fast-moving ice that erodes efficiently from sites affected by persistent slow-moving ice that does not erode efficiently.

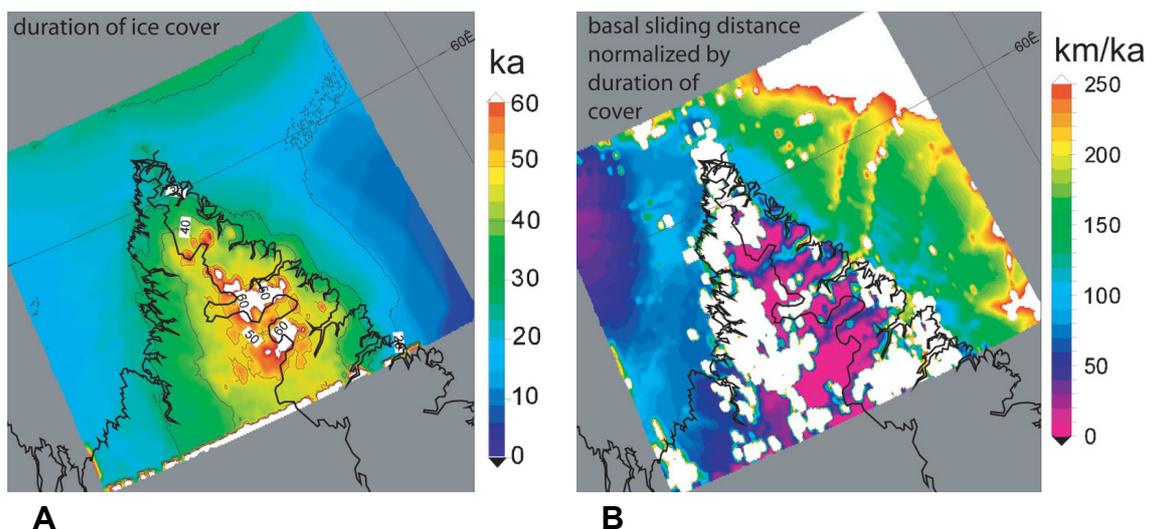


Fig. 4: UMISM results over a 100 ka cycle for the Labrador Peninsula, north-eastern Canada (taken from Staiger et al. 2005).

(A) Plan view of the duration (ka) of ice cover over the last glacial cycle. (B) Spatial variation in the modelled basal sliding distance normalized by duration of ice cover. The ice simulation shows that the plateau and peak surfaces were probably positioned below slow-moving, cold-based ice whereas lower altitude surfaces were overridden by faster thicker, warm-based ice.

Simulations for the Labrador Peninsula over a 100 ka cycle reveal the development of a local thin, slow-moving ice cover on summit surfaces preceding the formation of the Torngat Mountain ice cap as well as the transformation of the complete ice cover during glaciation to valley ice and small isolated summit ice caps and cirque glaciers. The patterns of basal ice-sliding distance normalized by the duration of ice cover (Fig. 4B) further show that in areas that are completely covered by polythermal glacier ice, differential rates of glacial erosion tend to

complement valley deepening. Because erosion in glaciated regions is controlled mostly by the basal thermal regime, Staiger et al. (2005) suggest that generation of relief in the Torngat Mountains is due to the juxtaposition of cold-based, non-erosive ice on plateau interfluves at high elevations and wet-based, faster-moving and thus erosive ice on deep valley floors.

### 4.3 Sediment fluxes beneath the Antarctic Ice Sheet

DeConto & Pollard (2003a, b) use a numerical ice-flow model coupled to a Global Climate Model (GCM) to simulate the early glacial history of Antarctica. The GCM component of the model is GENESIS (Pollard & Thompson 1997, Thompson & Pollard 1997), with specific modifications to produce realistic snowfall and melt, and thus to provide realistic surface mass-balance forcing for the three-dimensional ice sheet model. Pollard & DeConto (2003, 2007) further develop the climate-ice sheet model through the incorporation of a model of deforming sediment, allowing the erosion, transport and deposition of glaciogenic material to be predicted. The thermomechanical ice-sheet model is used to calculate the evolution of the ice sheet geometry under the influence of surface mass balance, basal melting and ice flow. Ice temperatures are predicted for their effect on ice rheology and basal conditions. The sediment component (Clark & Pollard 1998) added to the ice sheet model accounts for extra sliding due to horizontal shear in a layer of sediment, driven by the basal shear stress of the ice. A weakly non-linear sediment rheology (Jenson et al. 1995, 1996) allows significant shear in the upper section of the sediment layer, with deformation occurring only where the basal temperature is at the pressure melting point and the sediment is saturated. Two simple parameterizations are added to simulate the long-term continental-scale evolution of sediment budgets and fluxes: ice-free transport of sediment by rivers is modelled as instantaneous fluvial transport down the topographic gradient; and new sediment is generated subglacially, at a rate proportional to the work done by basal sliding, with the constant of proportionality increasing as the sediment thins.

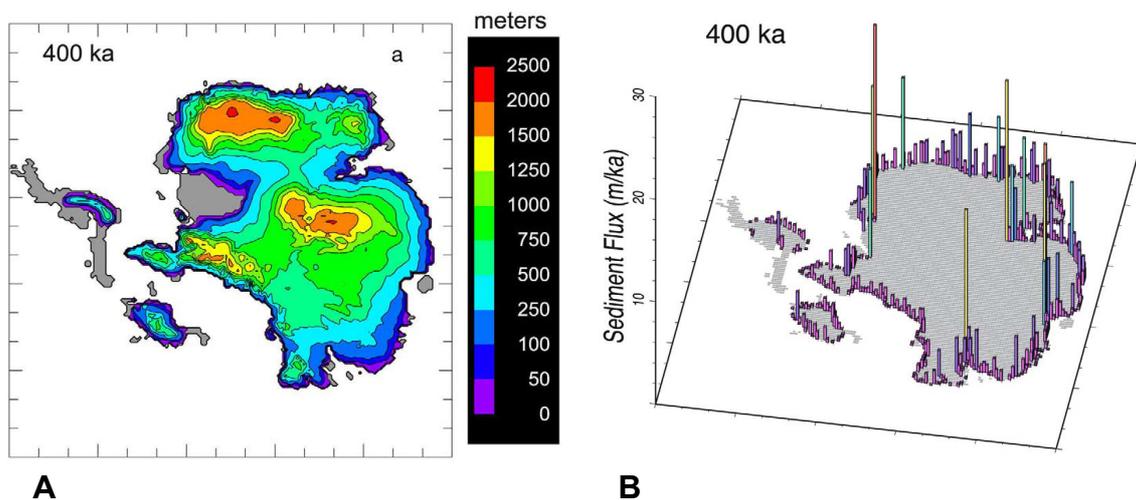


Fig. 5: Modelling sediment transport and deposition by the Antarctic Ice Sheet during a 400 ka period of its Eocene-Oligocene phase ( $\sim 34$  Ma BP) of orbitally-forced growth and decay cycles (taken from Pollard & DeConto 2003).

(A) Average ice thickness. (B) Average coastal sediment flux.

Model results reveal how the presence of deforming basal sediments cause the glacial-interglacial signal of ice sheet initiation, growth and retreat, to be amplified due to the reduction in basal drag and consequent increases in ice velocities. The sediment component of the model allows a determination of where to expect major deposits of material. Most of the sediment flux occurs at just a few coastal sites at the mouths of major topographic and bathymetric troughs (Fig. 5B).

#### 4.4 Evolution of the glacial landscape, Southern Alps, New Zealand

Herman & Braun (2008) apply a landscape evolution model to the Southern Alps of New Zealand to investigate the interplay between fluvial and glacial erosion under orogenic evolution during times of alternating glacial and interglacial periods. The model is a further development and refinement of the surface processes model ICE CASCADE (Braun et al. 1999; Tomkin & Braun 2002; Tomkin 2003, 2007) and combines ice sheet evolution with processes of glacial erosion, fluvial erosion and hill-slope development. Glacial erosion is simply taken to be a function of the ice sliding velocity, and the basal temperature, which determines whether sliding occurs, is calculated in a one-dimensional column model as the balance between vertical heat conduction and advection. Fluvial erosion is assumed proportional to the difference between the actual sediment load, calculated by integrating upstream erosion/deposition, and the stream carrying capacity expressed as linear function of slope and water discharge. Hill-slope processes are assumed to include a variety of processes such as weathering, slope wash, mass wasting and soil creep, and are incorporated in the model through a linear diffusion equation.

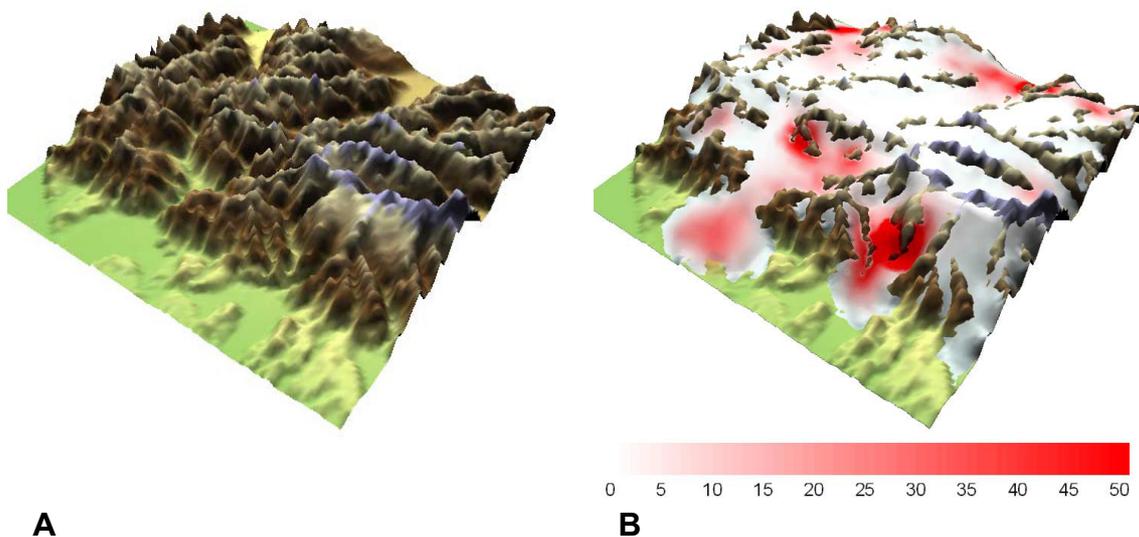


Fig. 6: Perspective view of the Southern Alps, New Zealand. North points to the left. The area shown is  $64 \times 64 \text{ km}^2$ . Vertical scale is exaggerated for clarity (taken from Herman & Braun 2008).

(A) Present-day topography. (B) Modelled ice extent, sliding velocity (in  $\text{m a}^{-1}$ ) and, by implication, erosion pattern at the Last Glacial Maximum (LGM).

Figure 6B shows the patterns of erosion induced by glaciers at the glacial maximum (noting that the colour of the ice depicts the basal sliding velocity and thus is proportional to erosion rate). Erosion is focused along the steep sections of the valleys and where the ice accumulates by

convergence of flow at the mouth of the main valleys. Because of large variations in ice geometry during a glacial cycle, the regions of the landscape where glacial erosion is most intense vary greatly through time. Indeed, the model simulations reveal that the sliding portion of the ice cover changes position as the ice extent evolves, in turn, inducing a complex time-varying erosion pattern. For ice erosion to take place, the basal temperature must be at the melting point. The basal temperature increases proportionally to ice thickness and surface temperature. Hence the part of the landscape where glaciers are actively eroding is a function of ice thickness and surface temperature and does not necessarily correspond to locations of the equilibrium line altitude (ELA) as suggested by, e.g., Anderson et al. (2006). When the ice extent is maximum, basal melting occurs at lower elevations where ice thickness and surface temperature are relatively high, which leads to the formation of large overdeepened regions extending for several kilometres into the forelands (Fig. 7B).

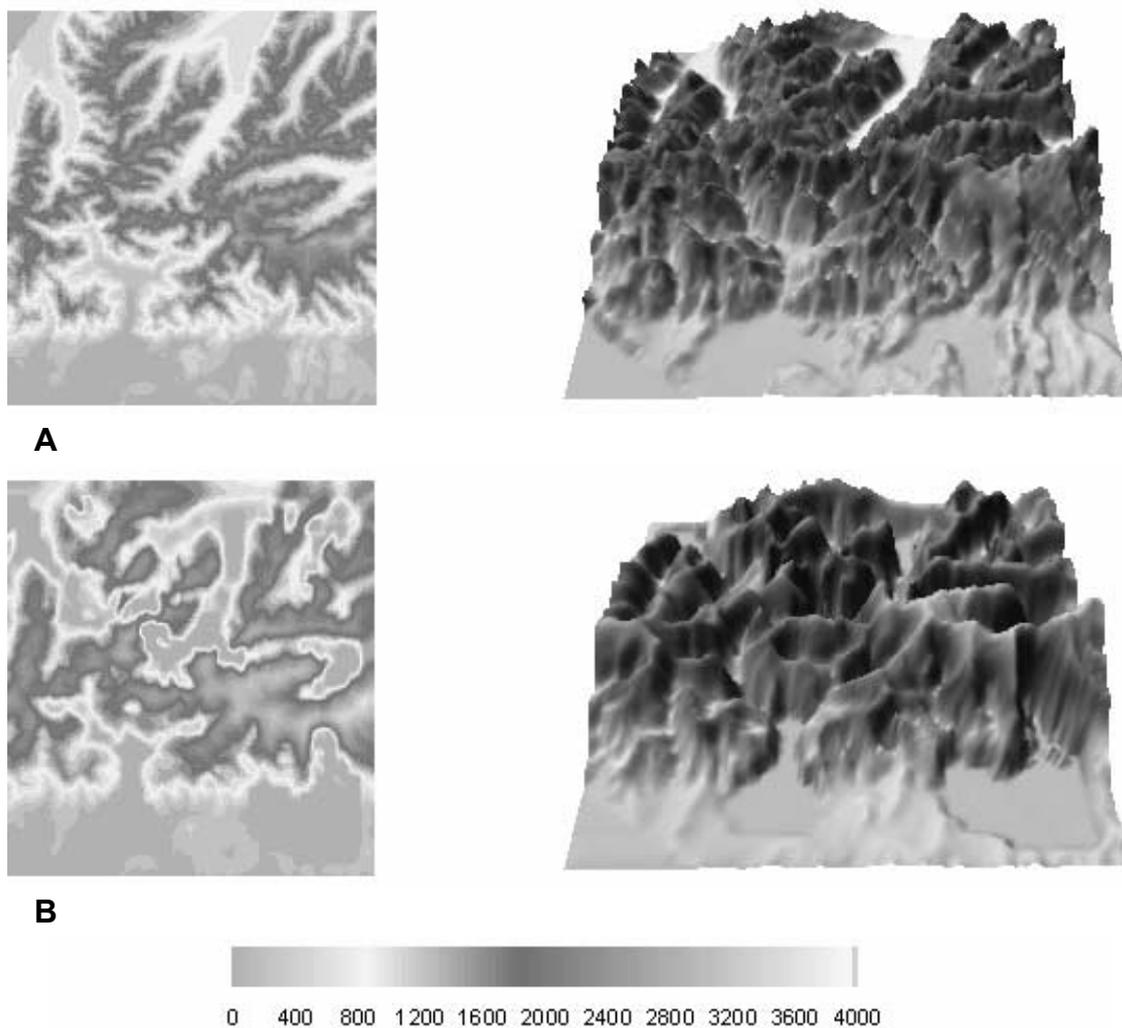


Fig. 7: Result from numerical experiments showing (left) plan views and (right) perspective views of the Southern Alps, New Zealand (taken from Herman & Braun 2008).

(A) Present-day topography. (B) Topography after one glacial cycle (120 ka).

#### **4.5 Subglacial erosion and englacial sediment transport, North American ice sheets**

Hildes et al. (2004) take a non-reductionist approach to the problem of calculating erosion rates by incorporating a comprehensive understanding of glacial sedimentary processes. Rather than simply relating the erosion of sedimentary material to basal motion, a physically-based model of basal processes comprises representations of subglacial erosion, sediment entrainment, englacial mixing, advective transport and deposition. Erosion is included by following and developing the work of Tulley (1995), which accounts for the process of ice sheet abrasion over a variety of bedrock lithologies and by the excavation of blocks through the quantification of crack propagation in basal rock. In this model the basal processes are dependent on ice sheet geometry, hydrology and dynamics. The UBC (University of British Columbia) three-dimensional thermomechanical ice sheet model (Marshall & Clarke 1997; Marshall et al. 2000, 2002), driven by prescribed climate fields, yields the ice geometry, flow and thermal conditions that are used by a model of subglacial hydrology. Subglacial water pressure, a critical input to the basal process model, is calculated using a hydrological model for coupled basal and groundwater drainage (Flowers & Clarke 2002).

In a detailed numerical investigation into subglacial sediment transport and distribution in glacial North America, Hildes et al. (2004) take a depiction of the surface geology (both hard rock and loose material) as input to the model to predict the development of sediment sources and sinks, and their effect on ice dynamics (Fig. 8). Despite breaking the problem of erosion into its process constituents, the results fail to correspond well with the known distribution of glacial material. Integrated erosion estimates are low because the model underestimates the vigour of the quarrying process.





## 5 Discussion

A full thermomechanical treatment of ice flow is advantageous in polythermal glaciers and ice sheets or when thermal conditions within an ice mass change, for example in response to changes in climatic conditions. A shortcoming of the model used by Herman & Braun (2008) is that it is not a true thermomechanical ice-sheet model and deals with temperature in a limited way when modelling erosion. In contrast, the models used by Jamieson et al. (2007), Staiger et al. (2005), Pollard & DeConto (2003) and Hildes et al. (2004) follow the established lineage of thermomechanical ice-sheet models that took part in the EISMINT model intercomparison (Huybrechts et al. 1996; Payne et al. 2000).

Unlike all other models described in this review, the ice-flow model used by Staiger et al. (2005) does not account for sediment erosion at the bed. The model is therefore not a coupled ice sheet/sediment model and cannot be applied to simulate processes of glacial erosion, transport and deposition. However, because the thermodynamic ice sheet model calculates normalized sliding distance it can be used to distinguish between zones of non-erosive ice and those of highly erosive ice which is an important consideration in the study of landform evolution beneath glaciers and ice sheets.

The modelling approaches of Jamieson et al. (2007) and Herman & Braun (2008) are limited by the fact that glacial storage and deposition of sediment is not included. Sediment generated by glacial erosion is either instantaneously removed from the system (Jamieson et al. 2007) or routed to the edge of the ice, where it is included in the calculations of fluvial sediment load and transported or deposited by the river network depending on whether or not the available sediment exceeds the stream carrying capacity (Herman & Braun 2008; Tomkin 2009). Therefore, glacially eroded overdeepenings are not filled by sediments at the end of a glacial period which has important consequences for the geometry of the resulting drainage patterns and the efficiency of fluvial processes during interglacial periods.

In the model used by Pollard & DeConto (2003) subglacial sediment transfer is accounted for by including a deforming sediment layer beneath the ice as the process of sediment deformation leads to a flux of material in the direction of ice flow. However, an appropriate rheology for subglacial sediment is critical to coupled ice-flow/sediment models because it not only dictates the rate of flow of sediment but also determines the large-scale ice flow dynamics. Field observations of sediment deformation suggest that basal sediments deform plastically (e.g. Clarke 2005), but incorporating such flow effectively into ice flow models is problematic, due to the nonlinearity between stress and strain rates.

The modelling results of Jamieson et al. (2007), Staiger et al. (2005) and Herman & Braun (2008) indicate that increased glacial erosion occurs at lower elevations where the ice is able to warm up allowing for basal melting and higher glacier flow velocities, with the ice at higher elevations remaining cold based and protective of the bed. While this finding explains the formation of deep valley floors and overdeepened valley longitudinal profiles typical of glacial landscapes, many questions still remain regarding the glacial erosion processes beneath the frontal reaches of glaciers that lead to significant valley overdeepening in the forelands far away from the mountains.

Almost all of the models described in this review employ a simple glacial sliding-dependent rule of subglacial erosion. MacGregor et al. (2009) provide some improvement by suggesting an erosion rule that acknowledges abrasion and quarrying to be distinct processes that operate most effectively under specific basal conditions. None of the models, however, account for the

process of subglacial meltwater erosion. This might partly be due to the fact that the complex flow of water underneath the ice is often not addressed in models of ice-sheet flow. A notable exception is the modelling by Hildes et al. (2004) who include physical representations of subglacial hydrology and basal processes. In this respect, their model is certainly the most comprehensive attempt to quantify glacial erosion, transport and deposition. Although the results failed to meet expectations, the model represents a benchmark in attempts to fully quantify glacial sedimentary processes, and is a new base from which further studies can be developed.

## 6 Conclusions

Three-dimensional coupled ice-sheet/sediment modelling significantly contributes to a better understanding of the spatial and temporal role of glacial erosion. Ice-sheet models continue to improve, both in terms of model physics and technical capabilities. Considerable progress is being made in coupling climate models with dynamic ice-sheet models and current models are able to predict spatial and temporal ice-sheet response to changes in environmental conditions with increasing confidence. Receding computational limitations offer the possibility of further increases in model sophistication and resolution in the years ahead.

Further advances are desirable in the incorporation of more appropriate physics and boundary conditions. In particular, the treatment of subglacial hydrology and basal processes remains problematic. The concerted effort to better account for glacial sedimentary processes and the development of subglacial hydrological models should lead to improvements in the next generation of coupled ice-sheet/sediment models, to help investigate the feedbacks between the thermomechanical regime of ice sheets, long-term changes in climate and the evolution of subglacial landforms at ice sheet scales.

With respect to the long-term safety of deep geological repositories in Switzerland, such models can be used to address a number of related questions regarding the rate and magnitude of erosion beneath glaciers that extend far into the midland areas of northern Switzerland (Swiss Plateau) during past and future glaciations:

- What factors are important for the spatial and temporal distribution of large-area denudation by subglacial abrasion versus linear down-cutting of subglacial meltwater channels?
- Does the advancing ice at the onset of glaciations exploit an existing valley network or carve out new valleys? Will lateral ridges remain essentially intact during future glacial advances?
- How does the resistance to erosion of different lithologies (consolidated rock, lake deposits, unconsolidated Quaternary deposits) affect erosion rates?
- What factors control the subglacial sediment balance? Where is sediment eroded, where is it transported to and where is it deposited? What are the relevant processes?
- How can the origin of deeply incised troughs and overdeepened valleys in the midland areas of northern Switzerland be explained? What processes are important for deep glacial erosion beneath the frontal reaches of glaciers far away from the Alps?
- Can the deeply incised troughs and overdeepened valleys in the midland areas of northern Switzerland essentially be cut in a single glacial cycle or does their formation require further deepening of existing valleys during subsequent glaciations?
- How are glacially eroded overdeepenings filled by sediments at the end of a glacial period?



## 7 References

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